Quarks and Gluons at Small x and Scaling Violation of F_2 [*]

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Abstract

I present some comments on the relationship between the small-x behaviour of the parton (quark and gluon) densities and the scaling violation of the proton structure function $F_2(x, Q^2)$.

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1 Introduction

The main point I shall consider in this contribution (a more detailed discussion can be found elsewhere [1]) is whether the striking rise of F_2, observed at HERA [2], calls for a theoretical interpretation in terms of non-conventional QCD dynamics. Here, non-conventional QCD stands for any approach (based either on the original BFKL equation [3] or on $k_\perp$-factorization [4, 5]) in which the small-x behaviour of $F_2(x, Q^2)$ is studied by resumming logarithmic corrections of the type $(\alpha_S \ln x)^n$ to *all orders* in the strong coupling α_S. By contrast, no small-x resummation is performed within the conventional QCD (or DGLAP [6]) approach: the parton densities of the proton at a fixed input scale Q_0^2 are evolved in Q^2 according to the Altarelli-Parisi equation evaluated in *fixed-order* perturbation theory.

It is certain that at asymptotically small values of x, the fixed-order expansion in α_S must become inadequate to describe the QCD dynamics. However, since the DGLAP approach successfully describes [7] the main features of HERA data, the signal of non-conventional QCD dynamics (at least from F_2, in the kinematic region explored at HERA so far) is *hidden* or *mimicked* by a strong background of conventional QCD evolution.

2 Scaling violation: the DGLAP approach

The master equations for the small-x behaviour of F_2 in perturbative QCD are (symbolically) as follows [1, 7]

$$F_2(x, Q^2) \sim \tilde{f}_S(x, Q^2) \ , \tag{1}$$

$$\frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} \sim P_{SS}(\alpha_S(Q^2), x) \otimes \tilde{f}_S(x, Q^2)$$

$$+ \ P_{Sg}(\alpha_S(Q^2), x) \otimes \tilde{f}_g(x, Q^2) \ , \tag{2}$$

where $P_{ab}(\alpha_S, x)$ are the (*calculable*) splitting functions and $\tilde{f}_S$ and $\tilde{f}_g$ are the (*phenomenological*) sea-quark and gluon densities.

The basis for Eqs. (1),(2) is provided by the the factorization theorem of mass singularities. According to this theorem, splitting functions and parton densities are not separately physical observables and, in particular, Eqs. (1),(2) refer to the so-called DIS scheme. However, when evaluated in two-loop order, the splitting functions slightly depends on the

factorization scheme and the HERA data can be succesfully described [7] by parton densities having the following small-x behaviour, $\tilde{f}_S(x, Q_0^2) \sim x^{-\lambda_S}$, $\tilde{f}_g(x, Q_0^2) \sim x^{-\lambda_g}$, with $\lambda_S = \lambda_g = 0.2 \div 0.3$ at the input scale $Q_0^2 \sim 4$ GeV^2. Actually, the HERA data may prefer [8] $\lambda_S \neq \lambda_g$, and, more precisely, $\lambda_S = 0.07 < \lambda_g = 0.3 \div 0.35$.

Up to the second order in α_S, the quark splitting functions P_{SS} and P_{Sg} in Eq. (2) are essentially flat at small x. Thus, the above results tell us that the rise of F_2 at small x is due to the DGLAP evolution in the gluon channel combined with a steep behaviour ($\sim x^{-0.3}$) of the input densities at $Q_0^2 \sim 4$ GeV^2. Moreover, taking seriously the results of the MRS(G) analysis [8], one can argue that $F_2(x, Q^2)$ is *not* very steep at Q^2-values of the order of few GeV^2 (see Eq. (1) with $\lambda_S = 0.07$) , but it is driven by *strong* scaling violations (see Eq. (2) with $\lambda_g = 0.35$).

Is there any room left for the non-conventional QCD approach? Is the power behaviour of the input parton densities (independently of the actual values of λ_S and λ_g) related to small-x resummation? Can we provide an explanation for the (possibly) favoured values $\lambda_S < \lambda_g$?

3 Scaling violation: small-x resummation

The above questions are formulated in the context of the parton picture and, as recalled in Sec. 2, the parton densities have a well-defined physical meaning only within the framework of the factorization theorem of mass singularities. Therefore, in order to answer to these questions, we have to relate the non-conventional QCD approach to the parton language.

A formalism which is able to combine *consistently* small-x resummation with the factorization theorem of mass singularities has been set up in the last few years [4, 5]. Within this formalism, known as $k_\perp$-factorization or high-energy factorization, one ends up with the usual QCD evolution equations (1) and (2), but the splitting functions $P_{ab}(\alpha_S, x)$ are no longer evaluated in fixed-order perturbation theory. They are indeed supplemented with the all-order resummation of the leading ($m = n - 1$), next-to-leading ($m = n - 2$) and, possibly, subdominant ($m < n - 2$) contributions of the type $\frac{1}{x}\alpha_S^n \ln^m x$ at small x. More importantly, this resummation can be performed by having full control of the factorization scheme dependence of splitting functions and parton densities.

One of the main outcome of these studies is the calculation [5] of the quark splitting functions P_{SS} and P_{Sg} to next-to-leading logarithmic accuracy in resummed perturbation theory. In particular, these resummed splitting functions, evaluated *in the* DIS *scheme*, turn out to be much steeper than their two-loop expansions in perturbation theory. Thus, *stronger scaling violations* at small x, were anticipated in Ref. [5].

Note that the *quark* splitting functions appear on the r.h.s. of the master equation (2). The large value of $\partial F_2(x, Q^2)/\partial \ln Q^2$ measured at HERA calls for a quite steep product (convolution) $P_{Sg} \otimes \tilde{f}_g$. In the DGLAP approach this condition can be fulfilled only by choosing a quite steep input distribution $\tilde{f}_g$. However, after resummation, $P_{Sg}(\alpha_S, x)$ has a small-x behaviour which is much steeper than that in two-loop order. Therefore, the use of resummed perturbation theory at small x may explain the scaling violations observed at HERA without the necessity of introducing a very steep input gluon density $\tilde{f}_g$. The results of recent numerical analyses [9] support this conclusion.

There is also an alternative (and more striking) way to restate the same conclusion on the possible relevance of small-x resummation for the HERA data on F_2. So far, I have only discussed the DIS scheme. One can consider a different factorization scheme, *the* SDIS *scheme* [1], in which the resummation effects discussed above are removed from the quark splitting functions and absorbed into the redefinition of the *gluon* density. In the new scheme, *i*) the resummed quark splitting functions differ slightly from the corresponding two-loop functions in the DIS scheme and *ii*) a steep gluon density (in particular, a gluon density steeper than the quark density) arises naturally as the result of small-x resummation. From the property i), it follows that the analysis of the scaling violations of F_2 in the SDIS scheme is very similar to that in the DGLAP approach. Thus, the property ii) offers a qualitative explanation of the results in Ref. [8]: the MRS(G) partons with $\lambda_g > \lambda_S$ may be interpreted as the partons in the resummed SDIS scheme.

4   Conclusion

In summary, HERA may have seen a (weak) signal of non-conventional small-x dynamics not in the (absolute) steep rise of F_2 but *rather* in stronger scaling violations at moderate values of Q^2. More definite conclusions demand further phenomenological investigations and more accurate data on F_2 in a

range of x and Q^2 as largest as possible.

Aknowledgements

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